Interdisciplinary Lively Applications Project

The Shuttle Problem

MATERIALS

1. Problem Statement (3 Situations); Student
2. Sample Solution; Instructor
3. Notes for the instructor

Computing Requirements:
A computer algebra system such as Maple, *Mathcad*, or *Mathematica*

Interdisciplinary Lively Application Project

Title: The Shuttle Problem

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Mathematical Classification: Vector Calculus, Differential Equations
Disciplinary Classification: Physics

Prerequisite Skills:
- Solving Ordinary Differential Equations using the method of Separation of Variables
- Differentiating Vector Functions
- Modeling Torque Using the Cross Product

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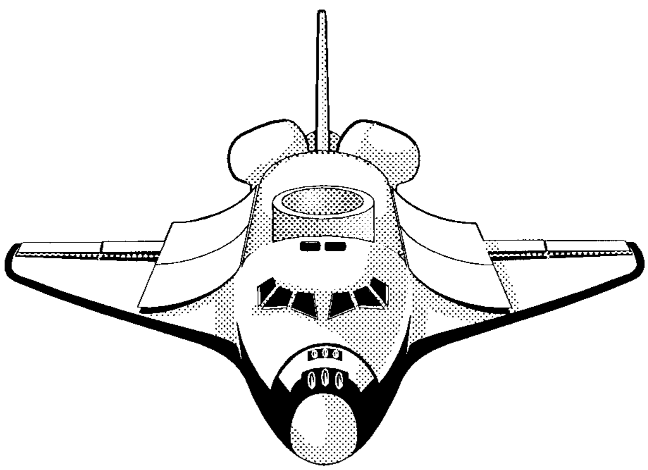
INTRODUCTION

NASA wishes to retrieve and repair a damaged satellite which is in danger of re-entering the Earth's atmosphere. NASA has called upon you to assist in various aspects of this upcoming shuttle mission. Not only will you be required to assist in the initial launch, but you will also be required to operate the robotic arm (Canadarm) in the retrieval process.



PART 1: ESCAPE SPEED

One of the first considerations for launching the shuttle is escape speed. This is the minimum initial speed at which a projectile must be launched in order to escape the Earth's gravitational field. For the shuttle, this is an upper bound for its launch speed as we do not want the shuttle to leave Earth's gravitational field. Since it is rather complicated to incorporate the changing rates of propulsion and thrust, in this initial analysis we treat the shuttle as a projectile — an object that is initially thrown, then has no inflight propulsion.

Newton's universal law of gravitation applied to this situation is given by

$F = -G\dfrac{M_e m_s}{r^2}$. Here, F represents the gravitational force exerted on the shuttle by the Earth, $G = 6.67\times10^{-11} N\dfrac{\text{m}^2}{\text{kg}^2}$ is a gravitational constant, $M_e = 5.98\times10^{24}\,\text{kg}$ is the mass of the Earth, m_s, is the mass of the shuttle (the shuttle weighs 220,000 pounds), and r is the distance from the shuttle to the center of the Earth. Additionally, the radius of the Earth is approximately 6.38×10^{8} meters.

A. Starting with Newton's Second Law of Motion, derive the shuttle's escape speed. (Hint: If the shuttle launches exactly at its escape speed, then in theory, when the velocity of the shuttle slows downs to zero, it has achieved infinite distance from the Earth. (You will also need to use the chain rule in order to determine that the derivative of the velocity, v, at any time t is given by $\dfrac{dv}{dt} = \dfrac{dv}{dr}\dfrac{dr}{dt} = \dfrac{dv}{dr}v$.

B. Comment on the feasibility of launching the shuttle at this speed; i.e., explain why it is or is not feasible, and what can be done if it is not feasible.

C. Now consider a more realistic scenario in which booster rockets are attached to the shuttle. If the booster rockets cease burning when the shuttle is 6000 miles above the Earth and the shuttle's speed at that time is 15,000 miles per hour, determine if the shuttle will escape the Earth's gravitational field.

D. Qualitatively, how does the mass of the shuttle affect the escape speed and the ability to get the shuttle into orbit? What are the implications of the shuttle burning fuel during flight?

PART 2: ORBIT ANALYSIS

NASA wants to know the acceleration experienced by the damaged satellite as it maintains a circular orbit. The satellite's position at any point (x,y) in the plane of the orbit can be modeled by the position vector $\overline{R}(t) = x(t)\hat{i} + y(t)\hat{j}$. However, since distances are measured relative to the center of the Earth and the orbit is circular, the use of polar coordinates simplifies computations. Thus, position will be measured in terms of angular and radial components.

A. Convert the position vector to its corresponding polar form. Note that the radius, $r(t)$, and angle $\theta(t)$, are functions of time.

B. Determine the associated acceleration vector.

C. Compute the acceleration in each direction (angular and radial) and write the acceleration vector as the sum of a radial and angular component. That is, write the acceleration vector you just computed in the form $\vec{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta$, where the unit radial vector is given by $\vec{u}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$,

   and the unit tangential (angular) vector is given by $\vec{u}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$.

   Hint: use the geometrical relationship between the two vectors $\vec{u}_r$ and $\vec{u}_\theta$.

D. Determine the acceleration vector for a satellite moving in a circular orbit 200 miles above the Earth with a constant angular speed of 4.1440 radians per hour. Interpret your results.

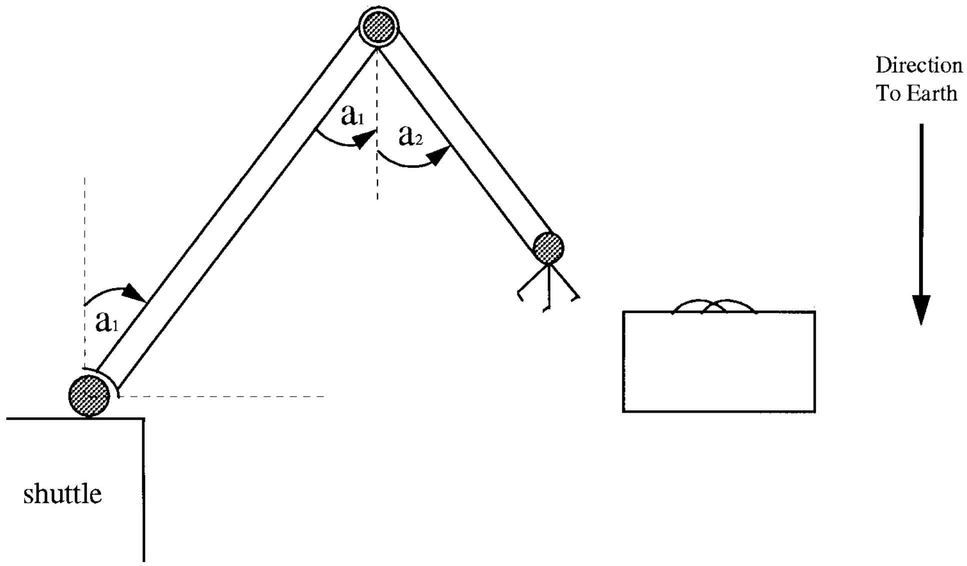
PART3: SATELLITE RETRIEVAL

Due to atmospheric drag, the satellite is losing altitude so quickly that it will burn up in the upper atmosphere unless it can be immediately placed into a higher orbit. NASA originally intended to place the satellite inside the shuttle bay and move it to a higher orbit. However, due to an onboard malfunction, most of the shuttle's breathable air has been lost. There is not enough time to retrieve the satellite, repair it, and then move it. NASA feels it is far too dangerous to have astronauts outside working on the satellite while the shuttle is maneuvering, so they plan to grab the satellite with the arm and move to a higher orbit while the satellite is being brought inside the bay. The manuever to a higher orbit requires a constant firing of the shuttle's thrusters, which gives the shuttle a constant acceleration of 0.3 m/sec^2, directed away from the Earth.

You are required to operate the robotic arm to retrieve the hapless satellite. From NASA's design specifications, the hinge that connects the robotic arm to the shuttle can only support a torque of 800*N*m. The arm itself consists of a long and a short portion, connected by a joint. There is a grappling hook at the end of the short portion that can be used to snag objects, such as the satellite, from orbit (ignore its length for this analysis). The long portion of the arm has length 9 m and mass 120 kg, and the short portion of the arm has length 7 m and mass 90 kg. The satellite weighs 500 pounds on Earth.

The satellite can be brought into the cargo bay when the two portions of the arm are vertical. It is your job to retrieve the satellite without allowing mechanical failure of the joint, i.e., without surpassing the torque restriction stated above. In a recent attempted docking maneuver, the MIR space station was damaged when a supply ship collided with it. In order to avoid a similar fiasco, NASA wants the shuttle to be as far away from the satellite as possible when it is retrieved.

There are two degrees of freedom to consider; the angle between the arm and the shuttle bay and the angle between the two portions of the arm. To simplify matters, model the angles of the arm as shown in the diagram below. Assume initially that the arm is geared in such a way that angles a_1 and a_2 are the same at all times. Since the material in the arm is uniform (same type of material, density, etc.) it is also reasonable to assume that the force exerted along the length of each portion of the arm is concentrated at the center of the respective portion.



A. What is the maximum distance the satellite can be from the shuttle yet still be retrieved?

B. Assuming that the shuttle can be moved to this distance, plot the return path of the satellite from the retrieval point to the cargo bay. Be sure to describe your reference system and any additional assumptions you make.

C. To make matters more realistic, now assume that the angles a_1 and a_2 are independent of each other. Derive an equation which expresses the magnitude of the torque as a function of the angles a_1 and a_2. Then, using a contour plot, graph the magnitude of the torque against the angles a_1 and a_2. Describe any overall trends that you observe .

SAMPLE SOLUTION

The following is a possible solution to the project. The software package, *Mathcad*, was used to generate this solution, and the *Mathcad* equations are distinguished by use of a different font. Using a package like *Mathcad* permits variation of key parameters between groups, permitting a range of possible solutions.

DESCRIPTION OF VARIABLES:

$m1{:=}120$	Mass of long portion of the arm, kg
$m2{:=}90$	Mass of the short portion of the arm, kg
$w1{:=}500$	Weight of satellite, lbs
$m3{:=}w1 \times 0.4535923$	
$m3=226.796$	Mass of the satellite, kg
$g1{:=}.3$	Acceleration of shuttle away from Earth, m/s^2
$L1{:=}9$	Length of long portion of arm, m
$L2{:=}7$	Length of short portion of arm, m
$t1{:=}800$	Maximum sustainable torque on arm hinge, *N*m
$G{:=}6.67 \times 10^{-11}$	Gravitational constant, *N*m^2/kg^2
$M_e{:=}5.98 \times 10^{24}$	Mass of the Earth, kg
$R_{\text{earth}}{:=}6.38 \times 10^6$	Radius of the Earth, m

PART 1:

A. We start with a description of forces from the free body diagram of the shuttle, and assume that the positive direction is away from Earth. There will only be one force, that of gravity. From Newton's Second Law,

$$m_s a = -GM_e m_s / r^2.$$

Use the given chain rule substitution, $a = dv/dt = (dv/dr)(dr/dt) = v\,(dv/dr)$, to get a differential equation we can solve:

$$m_s v(dv/dr) = -GM_e m_s / r^2.$$

Now divide by m_s and separate variables to obtain

$$v dv = -GM_e dr / r^2.$$

$$\int v\,dv \to \frac{1}{2} \times v^2$$

$$\int \frac{-G \times M_e}{r^2} dr \to G \times \frac{M_e}{r}$$

We get $\frac{1}{2} v^2 = \frac{GM_e}{r + C}$, where C is determined from initial conditions. Note that when $t = 0$, $v(0) = v0$ and $r(0) = R_{\text{earth}}$, the radius of the Earth. Thus,

$$C = v0 – GM_e/R_{\text{earth}}.$$

Solve for the variable r.

$$\frac{1}{2} \times v^2 = \frac{G \times M_e}{r} - \frac{G \times M_e}{R_{\text{earth}}} + \frac{1}{2} \times v0^2$$

$$r = 2 \times G \times M_e \times \frac{R_{\text{earth}}}{\left(v^2 \times R_{\text{earth}} + 2G \times M_e - v0^2 \times R_{\text{earth}}\right)}$$

The terms G, M_e, and R_{earth} are known constants, but we still need to determine the value of $v0$. We note that the above equation relates the variables r and v. We analyze the motion of an object leaving Earth to determine this unknown constant. As an object moves away from the Earth, its velocity decreases due to the Earth's gravity. However, if it is to completely escape Earth's gravitational field, its velocity must never decrease so much as to be negative. In other words, the minimum initial velocity required (the escape velocity) will have the property that as v goes to zero, r increases towards infinity. Hence, we require

$$\lim_{v \to 0} r = \lim_{v \to 0} \left(2GM_e \frac{R_{\text{earth}}}{v^2 R_{\text{earth}} + 2GM_e - v_0^2 R_{\text{earth}}} \right) \to \infty.$$

Thus, as v approaches zero, r will have a finite value unless the denominator approaches zero, i.e.,

$$0 = 2 \cdot G \cdot M_e - v0^2 \cdot R_{\text{earth}}$$

$$\text{or,} \quad v0 := \sqrt{\frac{2 \cdot G \cdot M_e}{R_{\text{earth}}}}$$

$$v0 = 1.118 \cdot 10^4 \text{ in m / s, or 25,014 miles per hour.}$$

B. It simply is not feasible to launch the shuttle at a speed of 11,180 m/s at the surface of the Earth. By way of comparison, the muzzle velocity of an M-16 rifle is only 1000 m/s. The shuttle would have to go from rest to the escape velocity nearly instantaneously. Any such acceleration would certainly kill the astronauts and destroy the shuttle. Given the large mass of the shuttle, and the very high speed required, this acceleration would also require tremendous force — far more than is possible.

C. The formula derived in part A for escape velocity is valid for any distance from the center of the Earth by simply replacing R_{earth} with r_d in the derivation, where r_d is the distance from the center of the Earth. Converting 6000 miles to meters and adding in the radius of the Earth, we get $r_d = 16.034 \times 10^6$ m. Applying our formula for escape velocity at this distance, we get $v_0 = 7053$ m/s. We compare this to the given speed of the shuttle $v_s = 15000$ mi/hr = 6704 m/s, and conclude that the shuttle will not escape Earth's gravity since $v_s < v_0$.

D. The mass of the shuttle, or any object for that matter, does not affect the escape speed, as the mass divides out in the derivation. However, the more massive the object, the more force is required to accelerate the object to orbital and escape velocities. The design and large mass of the shuttle make it extremely difficult to even get into high orbits, let alone reach escape velocity. According to NASA, the shuttle typically orbits between 100 and 330 miles above the Earth at velocities around 7800 m/s.

Part 2:

A. We want to convert the Cartesian equations into polar form. Convert them to vector notation so *MathCad* can do the work for us. The position vector now has the polar form $\begin{pmatrix} r(t)\cdot\cos(\theta(t)) \\ r(t)\cdot\sin(\theta(t)) \end{pmatrix}$, still using the standard rectangular basis vectors.

B. *MathCad* can easily calculate the necessary derivative for the acceleration vector

$$\begin{bmatrix} \frac{d^2}{dt^2} r(t)\cdot\cos(\theta(t)) \\ \frac{d^2}{dt^2} r(t)\cdot\sin(\theta(t)) \end{bmatrix} \to \begin{bmatrix} \left(\frac{d^2}{dt^2} r(t)\right)\cdot\cos(\theta(t)) - 2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\sin(\theta(t))\cdot\frac{d}{dt}\theta(t) - r(t)\cdot\cos(\theta(t))\cdot\left(\frac{d}{dt}\theta(t)\right)^2 - r(t)\cdot\sin(\theta(t))\cdot\frac{d^2}{dt^2}\theta(t) \\ \left(\frac{d^2}{dt^2} r(t)\right)\cdot\sin(\theta(t)) + 2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\cos(\theta(t))\cdot\frac{d}{dt}\theta(t) - r(t)\cdot\sin(\theta(t))\cdot\left(\frac{d}{dt}\theta(t)\right)^2 + r(t)\cdot\cos(\theta(t))\cdot\frac{d^2}{dt^2}\theta(t) \end{bmatrix}$$

C. In order to determine radial and angular components of this system, one can take scalar products with the radial vector, $\hat{u}_r = \begin{pmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{pmatrix}$, and the angular vector, $\hat{u}_\theta = \begin{pmatrix} -\sin(\theta(t)) \\ \cos(\theta(t)) \end{pmatrix}$.

However, if we group terms by sines and cosines, the following results:

$$\begin{bmatrix} \left[-2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\frac{d}{dt}\theta(t) - r(t)\cdot\frac{d^2}{dt^2}\theta(t)\right]\cdot\sin(\theta(t)) + \left[\frac{d^2}{dt^2} r(t) - r(t)\cdot\left(\frac{d}{dt}\theta(t)\right)^2\right]\cdot\cos(\theta(t)) \\ \left[2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\frac{d}{dt}\theta(t) + r(t)\cdot\frac{d^2}{dt^2}\theta(t)\right]\cdot\cos(\theta(t)) + \left[\frac{d^2}{dt^2} r(t) - r(t)\cdot\left(\frac{d}{dt}\theta(t)\right)^2\right]\cdot\sin(\theta(t)) \end{bmatrix}.$$

The astute observation is that this is really

$$\begin{bmatrix} 2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\frac{d}{dt}\theta(t) + r(t)\cdot\frac{d^2}{dt^2}\theta(t) \\ 2\cdot\left(\frac{d}{dt} r(t)\right)\cdot\frac{d}{dt}\theta(t) + r(t)\cdot\frac{d^2}{dt^2}\theta(t) \end{bmatrix}\cdot\begin{pmatrix} -\sin(\theta(t)) \\ \cos(\theta(t)) \end{pmatrix} + \begin{bmatrix} \frac{d^2}{dt^2} r(t) - r(t)\cdot\left(\frac{d}{dt}\theta(t)\right)^2 \\ \frac{d^2}{dt^2} r(t) - r(t)\cdot\left(\frac{d}{dt}\theta(t)\right)^2 \end{bmatrix}\cdot\begin{pmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{pmatrix}.$$

This can be rewritten as $\left(2\cdot\frac{d}{dt}r\cdot\frac{d}{dt}\theta+r\cdot\frac{d^2}{dt^2}\theta\right)\hat{u}_\theta+\left(\frac{d^2}{dt^2}r-r\cdot\left(\frac{d}{dt}\theta\right)^2\right)\hat{u}_r$,

which is the expression for the acceleration vector using polar basis vectors, as required. This formula can be found in many standard Calculus texts, (see, for example, *Calculus*, *2nd Edition* by Finney and Thomas, p.788 or *Multivariable Calculus* by Bradley and Smith, p. 706).

D. Since we are in a circular orbit with constant angular speed, r and $\frac{d}{dt}\theta$ are constants. Therefore $\frac{d}{dt}r$, $\frac{d^2}{dt^2}r$, and $\frac{d^2}{dt^2}\theta$ all equal zero. This simplifies the equation for acceleration considerably: $\hat{a}(t)=-r\cdot\left(\frac{d}{dt}\theta\right)^2\hat{u}_r$.

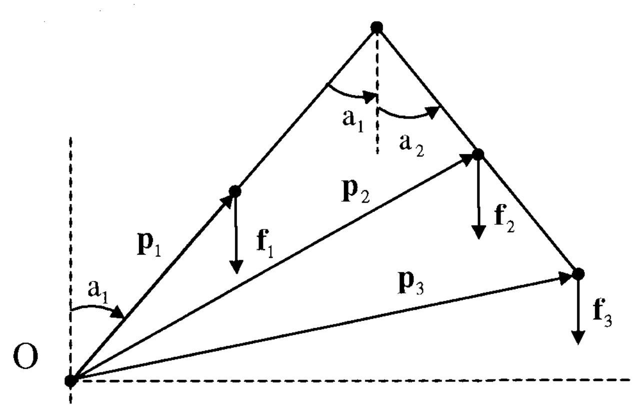
That is, all the acceleration is radial and constant. The negative sign indicates that the acceleration is directed towards the Earth. This is known as "centripetal acceleration", and is one of the consequences of the law of uniform circular motion. In this particular case, we are given that

$$r=R_{\text{earth}}+200\text{ mi.}=6.70\cdot10^6\text{m and }\frac{d\theta}{dt}=4.1440\text{ radians / hr}=0.001151\text{radians / sec},$$

which yields $\hat{a}(t)=-8.777\hat{u}_r$ in m/s^2. Physically, the radial acceleration makes sense, since the shuttle is in free fall and the only force acting on the shuttle is gravity.

Part 3:

A. In order to find the maximum distance that the shuttle can be from the satellite, we need to determine the magnitude of the torque on the hinge of the arm as a function of the angle a_1 . We can then set this magnitude equal to the torque restriction and solve for the angle which provides the maximum extension of the arm. It is then a simple calculation to determine the distance from the shuttle to the satellite.



We use the free body diagram above to find an explicit description of the moment arm vectors. Here we assume that the origin is at the middle of the hinge on the shuttle. Using right triangles and trigonometry we can determine the positions (p_1–p_3) where force is being applied.

$$p_1(a_1):=\begin{bmatrix}\frac{L_1}{2}\cdot\sin(a_1)\\ \frac{L_2}{2}\cdot\cos(a_1)\\ 0\end{bmatrix}\quad p_2(a_1,a_2):=\begin{bmatrix}L_1\cdot\sin(a_1)+\frac{L_2}{2}\cdot\sin(a_2)\\ L_1\cdot\cos(a_1)-\frac{L_2}{2}\cdot\cos(a_2)\\ 0\end{bmatrix}\quad p_3(a_1,a_2):=\begin{pmatrix}L_1\cdot\sin(a_1)+L_2\cdot\sin(a_2)\\ L_1\cdot\cos(a_1)-L_2\cdot\cos(a_2)\\ 0\end{pmatrix}$$

The following are the force vectors at each of the three points where the forces are applied, i.e., the middle of each arm portion and at the end of the arm where the satellite is attached. Note that the negative signs denote the forces are directed towards Earth.

$$f_1:=\begin{pmatrix}0\\ -m_1\cdot g_1\\ 0\end{pmatrix}\quad f_2:=\begin{pmatrix}0\\ -m_2\cdot g_1\\ 0\end{pmatrix}\quad f_3:=\begin{pmatrix}0\\ -m_3\cdot g_1\\ 0\end{pmatrix}$$

The corresponding torque equation is the magnitude of the cross products of the arm vectors and force vectors:

$$\text{torque}(a_1 a_2):=\left|p_1(a_1)\cdot f_1+p_2(a_1,a_2)\cdot f_2+p_3(a_1,a_2)\cdot f_3\right|$$

Initially we assume the angles a_1 and a_2 are equal. Thus, to find the value of a_1 which maximizes torque at the hinge, we need to find the root of torque(a_1,a_1) - t_1.

Initial guess : $a_1 := 0.4$

$$\text{maxangle} := \text{root}(\text{torque}(a_1,a_1)-t_1,a_1)$$

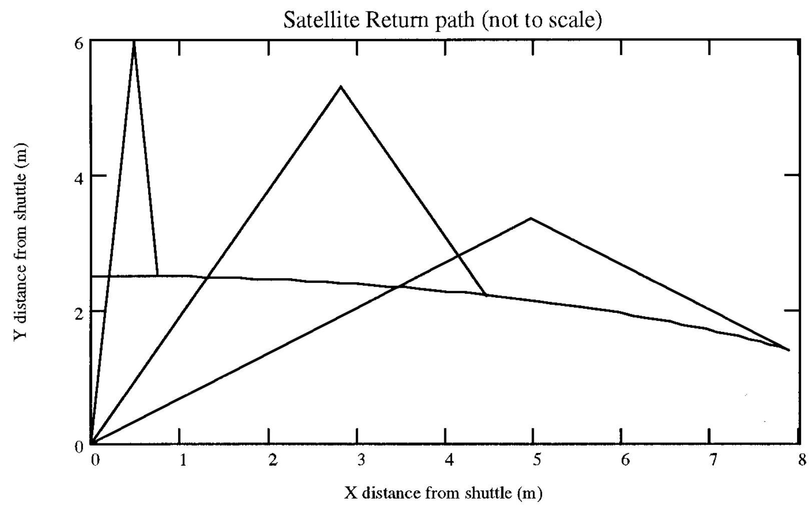
maxangle:= 0.528 in radians, or 30.25 degrees.

The maximum distance from the satellite, then, is the magnitude of p_3 at this angle, or $|p_3(a_1,a_1)| = 8.244$ m.

B. We now graph the return path of the satellite. Recall that the origin is the hinge on the shuttle. We start with the maximum angle, maxangle, and decrease to zero. We use a parametric plot, with $x(t)$ and $y(t)$ representing the coordinates of the end of the arm.

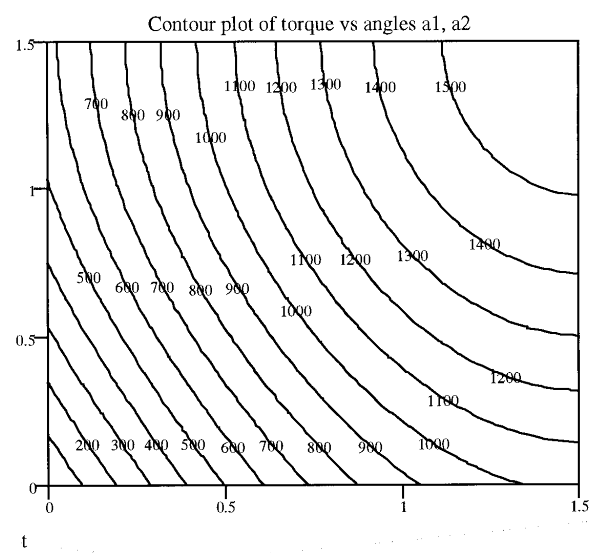
$$x(t):=(L_1+L_2)\cdot\sin(t)$$
$$y(t):=(L_1+L_2)\cdot\cos(t)$$

$$t:=\text{maxangle},\ \text{maxangle}-0.02..0$$



Note that these parametric equations represent an arc of an ellipse, shown as the curve in the above graph. The straight lines on the graph represent the position of the arm (not to scale) at three points along the path.

C. For the multivariable problem, we use a contour plot where the angles run from 0 to $\pi/2$. The torque equation is the same as in part B, however we no longer assume $a_1 = a_2$.



This contour plot shows how the angles a_1 and a_2 affect torque. Any pair of angles to the left of the 800 Nm contour is feasible for retrieving the satellite. The minimum torque occurs at the origin. Here both angles are zero and the arm is directly above the shuttle parallel to the acceleration vector. The maximum torque occurs at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$. This corresponds to full extension of the arm from the shuttle perpendicular to the acceleration vector. All masses are now the maximum horizontal distance from the hinge, maximizing the torque.

NOTES FOR THE INSTRUCTOR

We designed this project to show students that seemingly disparate ideas could be brought together to solve a larger problem. One part relied heavily on differential equations while the other two relied on vectors. This was sufficient for students to appreciate how several aspects of mathematics could be used together.

PART 1:

To help explain the concept of escape velocity to students, we used the following example. If we shoot a bullet in the air, it will slow down, stop, and then return to Earth. Shoot it faster, and it takes longer for the bullet to stop. Escape velocity is the minimum initial speed required to guarantee that the bullet will never return. Using the language of limits, "as the velocity approaches zero the distance from the center of the Earth approaches infinity." Not exactly rigorous, but it does give the right idea.

Naturally, there are several alternatives to the sample solution. One could also take $\lim_{r\to\infty}\left(\frac{1}{2}v^2 = GM_e/r + C\right)$ in order to show that C=0. Another alternative is to use definite integration and an improper integral, but the limits of integration need to be well explained. Also, if one chooses to use the energy considerations commonly used in many Physics courses, all that is required is to equate the decrease in kinetic energy with the rise in potential energy. By solving the resulting equations the escape velocity falls out (no pun intended).

The students may question why escape speed is considered for the shuttle since the shuttle will always be under the influence of gravity (while in orbit) during its missions. As stated in the project, the escape speed is an upper bound for the speed at which the shuttle can be launched. Our primary intent here was to introduce this concept and require students to apply their mathematical skills to derive it from Newton's Laws. Escape speed was a topic to be discussed later in their Physics course, relying on energy considerations for its derivation. However, by using Newton's Laws, the students had the additional benefit of being exposed to a problem involving a nonconstant acceleration which typically isn't included in most introductory physics courses.

PART 2:

Arbitrary angular velocities cannot be used in part 2. Orbital velocities should be used so that students may check that the acceleration due to gravity at the given altitude matches the acceleration on the shuttle that they calculate. The centripetal acceleration of an object in uniform circular motion is given by $a = \frac{v^2}{r}$, where r is the distance from the center of the Earth, and v is the tangential speed. For an object in orbital equilibrium, this acceleration should match the acceleration due to gravity from Newton's Universal Law of Gravitation.

One can greatly simplify the derivation for the polar form of the acceleration vector by assuming that r and $\frac{d\theta}{dt}$ are constant in each step. There are a couple of reasons we recommend that students not use these assumptions until they finish the general derivation. First, students can always use the extra practice with the chain rule. More importantly, there is rarely an orbit that has constant angular velocity or that is circular. Kepler's laws imply that both r and $\frac{d\theta}{dt}$ change in the more usual elliptical orbit. It may help to point out that each term in the acceleration vector has a physical interpretation: the radial, centripetal, tangential, and coriolis accelerations.

PART 3:

The condition that the shuttle is firing its engines while retrieving the satellite is necessary for a non-trivial torque problem. When the shuttle is in orbit, it is usually in free fall; its engines are not firing and it is in equilibrium. To an observer in the frame of reference of the shuttle, there seem to be no external forces acting on anything in the shuttle. With no forces, there can be no torque. It is for this reason that astronauts in orbit seem to be weightless, even though they experience centripetal acceleration due to gravity. To rectify this, assume the shuttle is firing its engines in such a way that it is moving away from the Earth with a constant acceleration, from a lower orbit to a higher one. This results in a "phantom gravity" pulling objects towards the Earth, and generates the force necessary for the torque analysis.

Vector products are not really necessary to find the torque since torque has both a scalar and vector definition. Although we preferred the vector approach for the sake of practice, students can find torque by using the horizontal component of the position vector to each point of mass and multiplying that distance by the force on that mass. It may be worthwhile to have students determine why this approach is equivalent to the vector method described in the sample solution. One could also use the Law of Cosines to arrive at a solution, but care must be taken as machine errors may develop in computing the solution.